

Harada-Tsutsui Gauge Recovery Procedure: From Abelian Gauge Anomalies to the Stueckelberg Mechanism

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Abstract

Revisiting a path-integral procedure of recovering gauge invariance from anomalous effective actions developed by Harada and Tsutsui, it is shown that there are two ways to achieve gauge symmetry: one already presented by the authors, which is shown to preserve the anomaly in the sense of standard current conservation law, and another one which is anomaly-free, preserving current conservation. It is also shown that the application of the Harada-Tsutsui technique to other models which are not anomalous but do not exhibit gauge invariance allows the identification of the gauge invariant formulation of the Proca model, also done by the referred authors, with the Stueckelberg model, leading to the interpretation of the gauge invariant map as a generalization of the Stueckelberg mechanism.

PACS numbers: 11.15.-q; 11.15.Tk; 11.30.-j

Keywords: gauge field theories; gauge anomalies; nonperturbative techniques

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I. INTRODUCTION

A gauge anomalous theory is one that presents a breakdown of gauge invariance at the quantum level [1]. When it happens, it is shown that the expectation value of the current divergence is not identically null, but, instead, there remains a term which is a function of the gauge field, which is called the anomaly. In this sense, it is used to be widely believed that there is a breaking of the current conservation law due to the presence of the gauge anomaly. This is one of the reasons why these models are not well liked, besides problems with renormalizability due to non-gauge invariance.

There is a range of contexts in which the discussion around anomalies is brought up, such as superstrings [2], quantum gravity [3, 4] and condensed matter phenomena description such as the fractional quantum Hall effect [5], for example. One of the fundamental prerequisites to renormalization and unitarity is the existence of the Stanislaw-Taylor identities, which seems to be spoiled by the presence of a gauge anomaly. For instance, one of the most important discussion was played by theories of Weyl fermions coupled to gauge fields, where the appearance of gauge anomalies is viewed as unavoidable, due to its quantum competition with chiral anomalies [6]. However, recently it was shown that when one goes to the full quantum level, where the gauge field is also quantized, then the expectation value of the anomaly must vanish [7]. For these reasons, it may be worth to re-discuss this subject in more detail.

In the eighties, an amount of discussion about anomalous models in quantum field theory was presented. The central role of discussion was played by consistence of such theories. Although some theorists considered such models as inconsistent, some authors produced works to support the idea that they are not so.

In this sense, we must cite the work of Jackiw and Rajaraman [8], in which it was shown that a gauge anomalous two-dimensional theory could be well defined and be able to provide a mass generation mechanism from chiral anomalies. This work was soon followed by the one of Fadeev and Shatashvili [9], who noticed that quantum gauge invariance could be restored by the introduction of new degrees of freedom that transform second class constraints into first class ones. In adding these extra fields, the effective anomalous action is mapped into a gauge invariant one. Then, the works of Babelon, Schaposnick and Viallet [10] and Harada and Tsutsui [11] showed independently that these degrees of freedom could emerge quite

naturally by the application of Faddeev-Popov's method through the non-factorization of the integration over the gauge group. Soon after, Harada and Tsutsui recognized that the same procedure could be applied to the Proca model [12], leading to possible generalization of their technique.

We can recognize the main strategy to give consistence to these models with the introduction of the new degrees of freedom, which recovers gauge invariance. In this sense, it seems useful to analyze such procedure and explore its potential. Although restoring gauge symmetry at the final effective action, one may ask whether such technique is able to provide current conservation or it just preserve the quantum anomaly.

This work is intended to elucidate this question for the particular case of abelian gauge anomalies. In this sense, in section II the origin of abelian gauge anomaly is briefly reviewed in path integral approach. In section III, the gauge invariant formalism developed by Harada and Tsutsui (HT) is rederived by redefining the vacuum functional multiplying it by the gauge volume, instead of proceeding with Faddeev-Popov's method, and it is shown that the anomaly is preserved in the original form proposed by the authors. Section IV is intended to show that their procedure gives rise to another abelian gauge invariant formulation which may provide an anomaly free model. In section V, the HT procedure applied to the Proca model is rederived. Finally, in section VI, a correspondence between the Proca's gauge invariant mapping and the Stueckelberg model is pointed out, leading to the interpretation of the HT procedure as a generalization of the Stueckelberg mechanism [13]. The conclusion is, then, presented in section VII.

II. THE ORIGIN OF ABELIAN GAUGE ANOMALY IN PATH INTEGRAL APPROACH

Consider an abelian gauge theory described by the action

$$I[\psi, \bar{\psi}, A_\mu] = I_M[\psi, \bar{\psi}, A_\mu] + I_G[A_\mu] \quad (1)$$

where $I_M[\psi, \bar{\psi}, A_\mu]$ is the matter action minimally coupled to the abelian gauge field A_μ , and $I_G[A_\mu]$ is the free bosonic action. If the action is said to be invariant under local gauge

transformations

$$\psi \rightarrow \psi^\theta = \exp(i\theta(x))\psi \quad (2)$$

$$\bar{\psi} \rightarrow \bar{\psi}^\theta = \exp(-i\theta(x))\bar{\psi} \quad (3)$$

$$A_\mu \rightarrow A_\mu^\theta = A_\mu + \frac{1}{e}\partial_\mu\theta(x), \quad (4)$$

one can say that, classically, the theory exhibits a conserved current given by

$$J^\mu = -\frac{1}{e}\frac{\delta I_M}{\delta A_\mu}. \quad (5)$$

Now, if we proceed the quantization of the fermionic fields, then, after integrating them out, we will arrive at an effective action give by

$$\exp(iW[A]) = \int d\psi d\bar{\psi} \exp(iI[\psi, \bar{\psi}, A]). \quad (6)$$

To find the quantum version of the current conservation law, first we make a change of variables in the fermion fields

$$\begin{aligned} \exp(iW[A]) &= \int d\psi d\bar{\psi} \exp(iI[\psi, \bar{\psi}, A]) \\ &= \int d\psi^\theta d\bar{\psi}^\theta \exp(iI[\psi^\theta, \bar{\psi}^\theta, A]), \end{aligned} \quad (7)$$

and then, just as in classical case, we make use of the invariance of the action by noticing that $I[\psi^\theta, \bar{\psi}^\theta, A] = I[\psi, \bar{\psi}, A^{-\theta}]$

$$\exp(iW[A]) = \int d\psi^\theta d\bar{\psi}^\theta \exp(iI[\psi, \bar{\psi}, A^{-\theta}]) \quad (8)$$

Now, a subtle difference between the classical and the quantum gauge theory arises: if the quantum measure is *locally* gauge invariant, *i. e.*, if

$$d\psi d\bar{\psi} = d\psi^\theta d\bar{\psi}^\theta, \quad (9)$$

then, by considering $\theta(x)$ as an infinitesimal parameter, we will have

$$\begin{aligned} \exp(iW[A]) &= \int d\psi^\theta d\bar{\psi}^\theta \exp(iI[\psi, \bar{\psi}, A^{-\theta}]) \\ &= \int d\psi d\bar{\psi} \exp\left(iI\left[\psi, \bar{\psi}, A_\mu - \frac{1}{e}\partial_\mu\theta(x)\right]\right) \\ &= \exp(iW[A]) - \int dx i\theta(x) \int d\psi d\bar{\psi} \partial_\mu \left(-\frac{1}{e}\frac{\delta I}{\delta A_\mu}\right) \exp(iI[\psi, \bar{\psi}, A_\mu]) \end{aligned} \quad (10)$$

$$\Rightarrow \int d\psi d\bar{\psi} \partial_\mu \left(-\frac{1}{e} \frac{\delta I}{\delta A_\mu} \right) \exp(iI[\psi, \bar{\psi}, A_\mu]) = 0. \quad (11)$$

But gauge invariance of the free bosonic action implies that $\partial_\mu \left(\frac{\delta I_G}{\delta A_\mu} \right) \equiv 0$, therefore,

$$\int d\psi d\bar{\psi} \partial_\mu \left(-\frac{1}{e} \frac{\delta I_M}{\delta A_\mu} \right) \exp(iI[\psi, \bar{\psi}, A_\mu]) = 0. \quad (12)$$

Equation (12) is the quantum version of the current conservation law. However, it was necessary to impose invariance of the fermionic measure (9) to get the above result. If, instead of (9), we had

$$d\psi^\theta d\bar{\psi}^\theta = \exp(i\alpha_1[A, \theta]) d\psi d\bar{\psi} \quad (13)$$

then, instead of (12), we would arrive at

$$\begin{aligned} \exp(iW[A]) &= \int d\psi^\theta d\bar{\psi}^\theta \exp(iI[\psi, \bar{\psi}, A^{-\theta}]) \\ &= \int d\psi d\bar{\psi} \exp(iI[\psi, \bar{\psi}, A^{-\theta}] + i\alpha_1[A, \theta]) \\ &= \int d\psi d\bar{\psi} \exp \left\{ iI[\psi, \bar{\psi}, A] + i \int dx \partial_\mu \theta(x) \left(-\frac{1}{e} \frac{\delta I}{\delta A_\mu} \right) \right. \\ &\quad \left. + \alpha_1[A, 0] + i \int dx \frac{\delta \alpha_1}{\delta \theta} \Big|_{\theta=0} \theta(x) \right\}, \end{aligned}$$

but $\partial_\mu \left(-\frac{1}{e} \frac{\delta I}{\delta A_\mu} \right) = \partial_\mu \left(-\frac{1}{e} \frac{\delta I_M}{\delta A_\mu} \right)$ and $\alpha_1[A, 0] = 0$, therefore

$$\begin{aligned} \exp(iW[A]) &= \int d\psi d\bar{\psi} \exp \left\{ iI[\psi, \bar{\psi}, A] - i \int dx \theta(x) \left[\partial_\mu \left(-\frac{1}{e} \frac{\delta I_M}{\delta A_\mu} \right) - \frac{\delta \alpha_1}{\delta \theta} \Big|_{\theta=0} \right] \right\} \\ &= \int d\psi d\bar{\psi} \exp(iI[\psi, \bar{\psi}, A]) \left\{ 1 - i \int dx \theta(x) \left[\partial_\mu \left(-\frac{1}{e} \frac{\delta I_M}{\delta A_\mu} \right) - \frac{\delta \alpha_1}{\delta \theta} \Big|_{\theta=0} \right] \right\} \\ &= \exp(iW[A]) - i \int dx \theta(x) \int d\psi d\bar{\psi} \exp(iI[\psi, \bar{\psi}, A_\mu]) \left[\partial_\mu \left(-\frac{1}{e} \frac{\delta I_M}{\delta A_\mu} \right) - \frac{\delta \alpha_1}{\delta \theta} \Big|_{\theta=0} \right] \\ &\Rightarrow \int d\psi d\bar{\psi} \partial_\mu \left(-\frac{1}{e} \frac{\delta I_M}{\delta A_\mu} \right) \exp(iI[\psi, \bar{\psi}, A_\mu]) = \mathcal{A} \exp(iW[A]), \end{aligned} \quad (14)$$

and we see that, instead of (12), we would have a non-vanishing right-hand side in (14), were

$$\mathcal{A} \equiv \frac{\delta \alpha_1}{\delta \theta} \Big|_{\theta=0} \quad (15)$$

is called the anomaly and the theory is said to be anomalous.

It is convenient, to our purposes, to rewrite the anomaly (15) by noticing that

$$\begin{aligned}
\left. \frac{\delta \alpha_1}{\delta \theta} \right|_{\theta=0} &= \left. \frac{\delta W[A^\theta]}{\delta \theta} \right|_{\theta=0} \\
&= \int d^n x \left(\frac{1}{e} \frac{\delta W[A]}{\delta A_\mu(y)} \right) \partial_\mu [\delta(x-y)] \\
&= \partial_\mu \left(\frac{1}{e} \frac{\delta W[A]}{\delta A_\mu(y)} \right),
\end{aligned}$$

and, therefore

$$\mathcal{A} \equiv \left. \frac{\delta \alpha_1}{\delta \theta} \right|_{\theta=0} = \partial_\mu \left(\frac{1}{e} \frac{\delta W[A]}{\delta A_\mu(x)} \right). \quad (16)$$

III. GAUGE INVARIANT FORMULATION OF ANOMALOUS MODELS

The anomaly arises from the non-invariance of the effective action. To see this, we notice that

$$\begin{aligned}
\exp(iW[A^\theta]) &= \int d\psi d\bar{\psi} \exp(iI[\psi, \bar{\psi}, A^\theta]) \\
&= \int d\psi^\theta d\bar{\psi}^\theta \exp(iI[\psi^\theta, \bar{\psi}^\theta, A^\theta]) \\
&= \int d\psi d\bar{\psi} \exp(iI[\psi, \bar{\psi}, A^\theta] + i\alpha_1[A, \theta]) \\
&= \exp(iW[A] + i\alpha_1[A, \theta]),
\end{aligned} \quad (17)$$

that is,

$$\Rightarrow \alpha_1[A, \theta] = W[A^\theta] - W[A]. \quad (18)$$

Therefore, from (14) it seems that current conservation at quantum level may be obtained only for theories with gauge invariant effective actions.

A gauge invariant formulation of anomalous theories was built by Harada and Tsutsui in [11]. We will derive the same results in a different way that is more convenient to our purposes, instead of inserting the usual Faddeev-Popov identity. It is considered the *full* theory, described by the vacuum functional

$$\begin{aligned}
Z &= \int d\psi d\bar{\psi} dA_\mu \exp(iI[\psi, \bar{\psi}, A]) \\
&= \int dA_\mu \exp(iW[A]).
\end{aligned} \quad (19)$$

The functional can be redefined by multiplying it by the gauge volume and, then, a change of variables in the gauge field can be performed

$$\begin{aligned} Z &= \int d\theta dA_\mu \exp(iW[A]) \\ &= \int d\theta dA_\mu^\theta \exp(iW[A^\theta]). \end{aligned} \quad (20)$$

Now we use the fact that the boson measure *is* gauge invariant, that is $dA_\mu = dA_\mu^\theta$, and we arrive at a theory containing a scalar field θ , besides the gauge field A_μ

$$\begin{aligned} Z &= \int d\theta dA_\mu \exp(iW'[A, \theta]) \\ &= \int dA_\mu \exp(iW_{eff}[A]), \end{aligned} \quad (21)$$

where

$$W'[A, \theta] \equiv W[A^\theta] \text{ and } \exp(iW_{eff}[A]) \equiv \int d\theta (iW'[A, \theta]) \quad (22)$$

It is easy to see that the new effective action $W_{eff}[A]$ is gauge invariant. To do this, we notice that

$$\begin{aligned} \exp(iW_{eff}[A^\lambda]) &= \int d\theta \exp(iW'[A^\lambda, \theta]) \\ &= \int d\theta \exp(iW'[A, \theta + \lambda]) \\ &= \int d(\theta + \lambda) \exp(iW'[A, \theta + \lambda]) \\ &= \exp(iW_{eff}[A]). \end{aligned} \quad (23)$$

One could ask if, after this procedure, the anomaly would survive, and we can say that it depends on the starting action. Indeed, one may choose an initial action by noticing that

$$\begin{aligned} Z &= \int d\theta dA_\mu \exp(iW'[A, \theta]) \\ &= \int d\theta dA_\mu \exp(iW[A^\theta]) \\ &= \int d\theta dA_\mu \exp(iW[A] + i\alpha_1[A, \theta]) \\ &= \int d\theta dA_\mu \exp(iI[\psi, \bar{\psi}, A] + i\alpha_1[A, \theta]). \end{aligned} \quad (24)$$

The action in eq. (24), with the addition of the Wess-Zumino term $\alpha_1[A, \theta]$ [15], is known as the standard action [11]

$$I_{st}[\psi, \bar{\psi}, A, \theta] = I[\psi, \bar{\psi}, A] + \alpha_1[A, \theta]. \quad (25)$$

As one could notice, although the final effective action $W_{eff}[A]$ is gauge invariant, the standard one $I_{st}[\psi, \bar{\psi}, A, \theta]$ is *not*, since $\alpha_1[A, \theta]$ breaks gauge invariance. To understand what it means, we see that, if we search for a kind of conserved current from this theory, we need to start from the gauge invariance of the effective action, which leads to

$$\partial_\mu \left(-\frac{1}{e} \frac{\delta W_{eff}[A]}{\delta A_\mu(x)} \right) = 0. \quad (26)$$

Then we have

$$\begin{aligned} & \partial_\mu \left(-\frac{1}{e} \frac{\delta W_{eff}[A]}{\delta A_\mu(x)} \right) \\ &= \frac{i}{e} \partial_\mu \left\{ \frac{\delta}{\delta A_\mu(x)} \exp(iW_{eff}[A]) \right\} \\ &= \frac{i}{e} \partial_\mu \left\{ \frac{\delta}{\delta A_\mu(x)} \int d\theta d\psi d\bar{\psi} \exp(iI_{st}[\psi, \bar{\psi}, A, \theta]) \right\} \\ &= \int d\theta d\psi d\bar{\psi} \partial_\mu \left(-\frac{i}{e} \frac{\delta I_{st}}{\delta A_\mu(x)} \right) \exp(iI_{st}[\psi, \bar{\psi}, A, \theta]) \\ &= \int d\theta d\psi d\bar{\psi} \partial_\mu \left(-\frac{i}{e} \frac{\delta I_M[\psi, \bar{\psi}, A]}{\delta A_\mu(x)} - \frac{i}{e} \frac{\delta \alpha_1[A, \theta]}{\delta A_\mu(x)} \right) \exp(iI_{st}[\psi, \bar{\psi}, A, \theta]) = 0, \end{aligned} \quad (27)$$

and since $\alpha_1[A, \theta]$ is not gauge invariant, one cannot say that $\partial_\mu \left(-\frac{1}{e} \frac{\delta \alpha_1[A, \theta]}{\delta A_\mu(x)} \right) = 0$, which would lead to the current conservation law. Instead, we have

$$\begin{aligned} & \int d\theta d\psi d\bar{\psi} \partial_\mu J^\mu \exp(iI_{st}[\psi, \bar{\psi}, A, \theta]) \\ &= \int d\theta d\psi d\bar{\psi} \partial_\mu \left(-\frac{1}{e} \frac{\delta \alpha_1[A, \theta]}{\delta A_\mu(x)} \right) \exp(iI_{st}[\psi, \bar{\psi}, A, \theta]) \neq 0. \end{aligned} \quad (28)$$

Now, we can perform integration over the θ field in the right-hand side of (28), using (13), (6) and the gauge invariance of $W_{eff}[A]$. It is straightforward to find

$$\int d\theta d\psi d\bar{\psi} \partial_\mu J^\mu \exp(iI_{st}[\psi, \bar{\psi}, A, \theta]) = \mathcal{A} \exp(iW_{eff}[A]), \quad (29)$$

and we see that the standard formulation still preserves the anomaly, in spite of being invariant at the effective theory. This may be explained by the switching of gauge symmetry breakdown from the effective action to the starting one, namely, the standard action.

IV. RECOVERING CURRENT CONSERVATION

The standard action is not the only one that can provide the gauge invariant effective theory given by (22). Indeed, from (21) we have

$$\begin{aligned} Z &= \int d\theta dA_\mu \exp(iW'[A, \theta]) \\ &= \int d\theta dA_\mu \exp(iW[A^\theta]) \\ &= \int d\theta d\psi d\bar{\psi} dA_\mu \exp(iI[\psi, \bar{\psi}, A^\theta]). \end{aligned} \quad (30)$$

Thus, we can see that the same procedure that leads to (21) and (6) can be done by a rather more obvious choice

$$I_{en}[\psi, \bar{\psi}, A, \theta] \equiv I[\psi, \bar{\psi}, A^\theta]. \quad (31)$$

which we can call, just to distinguish from the standard action, the *enhanced* action.

The advantage of this action is that it is really gauge invariant. Moreover, if we start from the gauge invariance of $W_{eff}[A]$ and proceed the same calculations which lead to (28), we will arrive at

$$\begin{aligned} &\partial_\mu \left(-\frac{1}{e} \frac{\delta W_{eff}[A]}{\delta A_\mu(x)} \right) \exp(iW_{eff}[A]) \\ &= \int d\theta d\psi d\bar{\psi} \partial_\mu \left(-\frac{1}{e} \frac{\delta I_{en}}{\delta A_\mu(x)} \right) \exp(I_{en}[\psi, \bar{\psi}, A, \theta]) \\ &= \int d\theta d\psi d\bar{\psi} \partial_\mu \left(-\frac{1}{e} \frac{\delta I[\psi, \bar{\psi}, A^\theta]}{\delta A_\mu(x)} \right) \exp(I_{en}[\psi, \bar{\psi}, A, \theta]) = 0 \end{aligned} \quad (32)$$

In fermionic theories, generally the gauge fields are coupled linearly to the fermions. So, expanding the matter action to the first order, we will obtain

$$\begin{aligned} I[\psi, \bar{\psi}, A] &= I_M[\psi, \bar{\psi}, A] + I_G[A] \\ &= I_F[\psi, \bar{\psi}] + \int d^n x \frac{\delta I_M[\psi, \bar{\psi}, A]}{\delta A_\mu(x)} + I_G[A], \end{aligned} \quad (33)$$

where $I_F[\psi, \bar{\psi}] \equiv I_M[\psi, \bar{\psi}, 0]$ corresponds to the free fermionic action. However $\frac{\delta I_M[\psi, \bar{\psi}, A]}{\delta A_\mu(x)} = -eJ^\mu(x)$, therefore

$$I[\psi, \bar{\psi}, A] = I_F[\psi, \bar{\psi}] + I_G[A] - e \int d^n x J^\mu(x) A_\mu(x) \quad (34)$$

$$I_M[\psi, \bar{\psi}, A] = I_F[\psi, \bar{\psi}] - e \int d^n x J^\mu(x) A_\mu(x) \quad (35)$$

Thus, evidently

$$-\frac{1}{e} \frac{\delta I_M [\psi, \bar{\psi}, A^\theta]}{\delta A_\mu(x)} = -\frac{1}{e} \frac{\delta I_M [\psi, \bar{\psi}, A^\theta]}{\delta A_\mu^\theta(x)} = -\frac{1}{e} \frac{\delta I_M [\psi, \bar{\psi}, A]}{\delta A_\mu(x)} = J^\mu(x). \quad (36)$$

Since $I_G[A]$ is gauge invariant, which means that $\partial_\mu \left(-\frac{1}{e} \frac{\delta I_G}{\delta A_\mu(x)} \right) \equiv 0$, eq. (32) leads to

$$\partial_\mu \left(-\frac{1}{e} \frac{\delta W_{eff}[A]}{\delta A_\mu(x)} \right) \equiv 0 \Leftrightarrow \int d\theta d\psi d\bar{\psi} \partial_\mu J^\mu(x) \exp(i I_{en} [\psi, \bar{\psi}, A, \theta]) \equiv 0. \quad (37)$$

Eq. (37) means that the current is conserved in this version of HT construction, with no quantum breakdown and, thus, anomaly-free.

To finish this section, we shall analyze the *classical* equations of motion obtained from the original abelian anomalous models

$$\frac{\delta I [\psi, \bar{\psi}, A_\mu]}{\delta \psi} = \frac{\delta I_M [\psi, \bar{\psi}, A_\mu]}{\delta \psi} = 0 \quad (38)$$

$$\frac{\delta I [\psi, \bar{\psi}, A_\mu]}{\delta \bar{\psi}} = \frac{\delta I_M [\psi, \bar{\psi}, A_\mu]}{\delta \bar{\psi}} = 0 \quad (39)$$

$$\frac{\delta I}{\delta A_\mu} = \frac{\delta I_M}{\delta A_\mu} + \frac{\delta I_G}{\delta A_\mu} = 0 \quad (40)$$

and compare them with those from the enhanced action $I_{en} [\psi, \bar{\psi}, A, \theta] \equiv I [\psi, \bar{\psi}, A^\theta]$

$$\frac{\delta I [\psi, \bar{\psi}, A_\mu^\theta]}{\delta \psi} = \frac{\delta I_M [\psi, \bar{\psi}, A_\mu^\theta]}{\delta \psi} = 0 \quad (41)$$

$$\frac{\delta I [\psi, \bar{\psi}, A_\mu^\theta]}{\delta \bar{\psi}} = \frac{\delta I_M [\psi, \bar{\psi}, A_\mu^\theta]}{\delta \bar{\psi}} = 0 \quad (42)$$

$$\frac{\delta I [\psi, \bar{\psi}, A_\mu^\theta]}{\delta A_\mu(x)} = \frac{\delta I_M [\psi, \bar{\psi}, A_\mu^\theta]}{\delta A_\mu(x)} + \frac{\delta I_G [A_\mu^\theta]}{\delta A_\mu(x)} = \frac{\delta I_M [\psi, \bar{\psi}, A_\mu]}{\delta A_\mu(x)} + \frac{\delta I_G [A_\mu]}{\delta A_\mu(x)} = 0 \quad (43)$$

$$\frac{\delta I}{\delta \theta} = \partial_\mu \left(-\frac{1}{e} \frac{\delta I_M [\psi, \bar{\psi}, A]}{\delta A_\mu(x)} \right) = \partial_\mu J^\mu = 0 \quad (44)$$

As one could see, the equation (44) for θ is redundant, since it is just the current conservation law imposed by global gauge invariance. The equation of motion for the gauge field is the same in both theories, since it is gauge invariant. Finally, the equations for the fermionic fields are reducible one to the other by a simple redefinition of the gauge field which is nothing but a generic gauge transformation $A_\mu \rightarrow A'_\mu = A_\mu + \frac{1}{e} \partial_\mu \theta$ that does not change the other equations. Thus, classically both formulations are completely equivalent one to the other, and the scalar is not even noted. On the other hand, at quantum level,

the simple original theory is anomalous, while the enhanced one, with the addition of the θ -field is not.

In the next sections, we shall understand why the θ - field can be absorbed with no loss of physical meaning by other means.

V. HT GAUGE RECOVERING PROCEDURE APPLIED TO NON-ANOMALOUS THEORIES - THE PROCA MODEL

As shown by the authors in the work of ref. [12] to the case of the massive vector field, the procedure of turning a theory that does not exhibit quantum gauge symmetry into a gauge invariant one does not need to be restricted to the particular class of gauge anomalous models. Indeed, to do so, it was only necessary to consider the exponential of the effective action $\exp(iW[A])$, gauge transform it into $\exp(iW[A^\theta])$, and then to perform an integration over θ to obtain, finally, the exponential of the gauge invariant effective action $\exp(iW_{eff}[A])$. But *any* action that does not exhibit gauge invariance could, in principle, be attached to this procedure. Let us reconsider, for instance, the massive vector field interacting with fermions, whose action is

$$I[\psi, \bar{\psi}, A_\mu] = I_M[\psi, \bar{\psi}, A_\mu] - \frac{1}{4} \int d^4x F^{\mu\nu} F_{\mu\nu} + \frac{m^2}{2} \int d^4x A^\mu A_\mu. \quad (45)$$

Clearly, the massive term breaks gauge invariance. If we consider the quantum version of this model and proceed with the HT technique, we will get

$$\begin{aligned} \int d\theta \exp(iW[A^\theta]) &= \int d\theta \exp(iI[\psi, \bar{\psi}, A^\theta]) \\ &= \int d\theta d\psi d\bar{\psi} \exp\left(I_M[\psi, \bar{\psi}, A_\mu^\theta] - \frac{1}{4} \int d^4x F^{\mu\nu} F_{\mu\nu} + \frac{m^2}{2} \int d^4x A^{\theta\mu} A_\mu^\theta\right) \\ &= \int d\theta d\psi^\theta d\bar{\psi}^\theta \exp\left(I_M[\psi^\theta, \bar{\psi}^\theta, A_\mu^\theta] - \frac{1}{4} \int d^4x F^{\mu\nu} F_{\mu\nu} + \frac{m^2}{2} \int d^4x A^{\theta\mu} A_\mu^\theta\right) \end{aligned} \quad (46)$$

and if the theory is not anomalous, that is, if $d\psi d\bar{\psi} = d\psi^\theta d\bar{\psi}^\theta$, we will arrive at an enhanced model given by

$$\exp(iW_{eff}[A]) = \int d\theta d\psi d\bar{\psi} \exp(iI_{en}[\psi, \bar{\psi}, A, \theta]), \quad (47)$$

where

$$I_{en} [\psi, \bar{\psi}, A_\mu, \theta] = I_M [\psi, \bar{\psi}, A_\mu] + \int d^4x \left(-\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \frac{1}{2} \frac{m^2}{e^2} \partial^\mu \theta \partial_\mu \theta + \frac{1}{2} m^2 A^\mu A_\mu + \frac{1}{2} \frac{m^2}{e} A^\mu \partial_\mu \theta \right). \quad (48)$$

If we proceed integration over the gauge parameter, we will find

$$\begin{aligned} \int d\theta \exp \left\{ \frac{i}{2} m^2 \int dx \left(\frac{2}{e} A_\mu \partial^\mu \theta + \frac{1}{e^2} \partial_\mu \theta \partial^\mu \theta \right) \right\} &= \exp \left(-\frac{1}{2} m^2 \int dx A_\mu \frac{\partial^\mu \partial^\nu}{\square} A_\nu \right) \\ \times \int d\theta \exp \left\{ i \frac{m^2}{2e} \int dx \left[\left(\frac{e}{\square} \partial^\mu A_\mu + \theta \right) \square \left(\frac{e}{\square} \partial^\nu A_\nu + \theta \right) \right] \right\}. \end{aligned} \quad (49)$$

Performing the change of variables $\theta \rightarrow \theta' = \theta + \frac{1}{\square} \partial^\mu A_\mu$; $d\theta' = d\theta$, we will arrive at

$$\int d\theta \exp \left\{ \frac{i}{2} m^2 \int dx (2A_\mu \partial^\mu \theta + \partial_\mu \theta \partial^\mu \theta) \right\} \sim \exp \left(-\frac{i}{2} m^2 \int dx A_\mu \frac{\partial^\mu \partial^\nu}{\square} A_\nu \right). \quad (50)$$

Using this result into (47), we finally obtain

$$\int d\theta d\psi d\bar{\psi} \exp (i I_{en} [\psi, \bar{\psi}, A_\mu, \theta]) = \int d\psi d\bar{\psi} \exp (i I' [\psi, \bar{\psi}, A_\mu]) \quad (51)$$

with

$$I' [\psi, \bar{\psi}, A_\mu] = I_M [\psi, \bar{\psi}, A_\mu] + \int d^4x \left\{ -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \frac{1}{2} m^2 A_\mu \left(\eta^{\mu\nu} - \frac{\partial^\mu \partial^\nu}{\square} \right) A_\nu \right\}. \quad (52)$$

It is easy to see that, classically, the gauge invariant formulation of Proca model (52) may be thought as equivalent to its correlate (45), since one is reducible to the other, with no loss of physical meaning, by the *Lorenz* gauge choice $\partial_\mu A^\mu = 0$. Therefore, this example clearly shows that the HT technique of inserting a quantum scalar may be used as a procedure to map a theory with no gauge symmetry into a gauge invariant one even in some cases where we are dealing with classical models.

VI. THE ENHANCED FORMALISM AND THE STUECKELBERG MECHANISM

In the enhanced gauge invariant formalism of anomalous models, we start with a gauge invariant action $I_{en} [\psi, \bar{\psi}, A, \theta]$, and reach an affective one $W_{eff}[A]$ which is also gauge invariant. However, there is an intermediate action $W'[A, \theta] = W[A^\theta]$ with no gauge symmetry.

Nevertheless, it is obviously invariant under a kind of expanded gauge transformations

$$\begin{aligned} A_\mu &\rightarrow A_\mu + \frac{1}{e} \partial_\mu \Lambda(x) \\ \theta &\rightarrow \theta - \Lambda(x) \end{aligned} \quad (53)$$

It means that we can set $\theta(x) = \text{constant}$ by a simple gauge choice and get back to the original formalism. In other words, classically, θ is not noted, but must exist and be quantized in order to provide an anomaly-free model. In section 4 we saw that the classical equations of motion of the enhanced version of anomalous models are reducible to those of the original one by a simple redefinition of the gauge boson. By the modified gauge symmetry (53) above, thus, it simple means a gauge choice where the scalar is set constant. On the other hand, the pure enhanced Proca model, which is also invariant under (53), is described by

$$I_P[A, \theta] = \int d^n x \left(-\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \frac{1}{2} \frac{m^2}{e^2} \partial^\mu \theta \partial_\mu \theta + \frac{1}{2} m^2 A^\mu A_\mu + \frac{m^2}{e} A^\mu \partial_\mu \theta \right). \quad (54)$$

If we simply redefine the θ - *field* by a multiplicative constant

$$B(x) \equiv \frac{m}{e} \theta(x), \quad (55)$$

then we will just find the Stueckelberg action [13]

$$I_{Stueck}[A, B] = \int d^n x \left\{ -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \frac{1}{2} (mA^\mu + \partial^\mu B)(mA_\mu + \partial_\mu B) \right\}, \quad (56)$$

and (53) becomes Pauli's gauge transformations [14]

$$\begin{aligned} A_\mu &\rightarrow A_\mu + \partial_\mu \Lambda(x) \\ B &\rightarrow B - m\Lambda(x). \end{aligned} \quad (57)$$

Therefore, we see that the HT procedure, using the enhanced form in the case of abelian models, is in closed connection with the Stueckelberg mechanism *before* the integration over the scalar, and may be viewed as its generalization, whose prescription is to attach a gradient of a scalar added up to every gauge boson.

The advantage of the Stueckelberg massive abelian model, which coincides exactly with the HT gauge invariant procedure applied to the Proca model before the integration over the extra degree of freedom, is that it was rigorously proved to be renormalizable and unitary [16].

We started by the integration over what we called the gauge parameter, but now we can reinterpret it by saying that it is not the gauge parameter which is actually integrated, but the Stueckelberg scalar. The procedure presented above shows that the Stueckelberg field can appear quite naturally, by going to the quantum level, and performing analytical manipulations over the functional integral that actually reveal it, instead of using the rather artificial Stueckelberg trick by adding the extra degree of freedom classically by hand. At the end, the Stueckelberg trick may be justified under this approach. We can interpret the Stueckelberg scalar as a hidden field, which is physically non observable at this point, but becomes necessary whenever we deal with gauge symmetry breaking and, as the mentioned examples, want to be able to provide a gauge anomaly-free theory as well as a renormalizable massive vector model.

VII. CONCLUSION

Revisiting a procedure to transform effective actions of anomalous generic models into gauge invariant ones, built in the last century, it was found that it can be more fruitful than it might have seem to be at a first sight. Indeed, the HT procedure is not only able to map an anomalous model into a gauge invariant one, but it may also be able to remove abelian gauge anomalies, which simply disappear when the θ -*field* is introduced into the theory by gauge transforming the gauge field. Moreover, it provides a bridge between the gauge invariant formulation of gauge anomalous models and a generalization of the Stueckelberg procedure, where the θ -*field*, identified as the Stueckelberg scalar, may be present together with the gauge field in any abelian theory, instead of being present only in the particular case of the massive vector model. The Stueckelberg mechanism was also extended to non-abelian gauge models, as can be seen in [17]. Perhaps the HT procedure, which was originally proposed for Yang-Mills models [11], may also be linked to this extension in the same way.

On the other hand, such discussion may raise a paradox: If one formalism is mapped into another one by simple manipulations over the functional integral, which would suggest that both formalisms are equivalent, how, in the anomalous case, one might present current conservation breakdown while the other has it conserved? As we have seen, the original formalism is anomalous, which would mean that it is closer to the standard formalism, that preserves its anomaly, then to the enhanced one. In this sense, one might ask which of

the two gauge invariant models is equivalent to the original one. This question may be partially answered in [7] where it was shown that the original anomalous formalism has the expectation value of the anomaly cancelled out when one goes to the full quantum theory, *i. e.*, the one with the gauge fields also quantized. Work is in progress to clarify this question in more detail.

The relevance of the Stueckelberg mechanism is that it is able to deal with gauge symmetry breaking and, since it is renormalizable, it provides a mechanism alternative to the Higgs [18]. Moreover, it can be recovered in a rather singular limit of the Higgs mechanism [19]. Therefore, perhaps the uncovered hidden scalar field might be regarded as an inheritance of the Higgs mechanism at lower energies. In revealing a generalization of the Stueckelberg mechanism, we saw that it is also able to provide a gauge anomaly-free model. On the other hand, it is well known, for the simplest case of the anomalous Jackiw-Rajaraman model, that there is an alternative mass-generation mechanism to the gauge boson from quantum corrections of anomalous $2\text{-}D$ chiral fermions [8]. Perhaps it is not mere coincidence that a breaking in the gauge symmetry in both cases is related to vector boson mass generation, and it may be recovered by an introduction of a scalar.

We can point out that, besides the correspondence between the HT procedure and the Stueckelberg mechanism, this technique might be generalized to other kind of symmetries, although it has been remarked that it may not be able to deal with chiral anomalies, for example [7]. It is well known that the Stueckelberg trick can also be used to restore gravitational gauge symmetry to deal with massive gravity models, for instance [20]. One might ask whether it would be justified by a kind of the prescription presented above, as it was shown for vector models. Finally, yet with the gravitational example, this procedure might be a road to cancel the gravitational anomaly, presented by the famous work of Witten and Gaumé [3]. In this sense, it was shown that the Hawking radiation can be explained by the addition of chiral fermions at the boundary of a black hole, that cancels the gravitational anomaly [4]. Perhaps it could happen in a more natural way, using the HT prescription adapted to the gravitational case, by substituting the anomalous chiral fermions by Stueckelberg fields.

Acknowledgments

I would like to thank Prof. A. J. Accioly and Prof. José A. Helayël Neto for further revisions. I also thank my colleagues Ricardo Kullock and Ricardo Scherer for constructive discussions about the subject of this work. Finally, I am very grateful to Professor Abhay Ashtekar and the *Institute for Gravitation and the Cosmos* (Penn State University) for the hospitality. This work was financially supported by CNPq through the Csf program (Ciência sem Fronteiras) from Brazil and in part by the NSF grant PHY 1205388 and the Eberly research funds of Penn State University.

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